Finite Differences

Use finite differences to determine the degree of the polynomial function that will fit the data.

<table>
<thead>
<tr>
<th>a)</th>
<th>x</th>
<th>0 1 2 3 4 5</th>
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<tbody>
<tr>
<td>f(x)</td>
<td>5 9 25 65 141 265</td>
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</table>

3rd degree cubic

2. Show that this function has constant second-order differences \( f(n) = \frac{1}{2} (n^2 + n) \).

3. Use finite differences to find the degree of the polynomial that will fit the data. 
\( f(1) = 1, f(2) = 4, f(3) = 10, f(4) = 20, f(5) = 35, f(6) = 56 \)

3rd degree cubic

4. Show that the fourth-order differences for the function \( f(x) = x^4 + x - 2 \) are nonzero and constant.

Average Rate of Change and Analyzing Polynomials

5. Without a calculator graph the function \( f(x) = \frac{1}{4} (x + 2)(x - 1)^2 \) if it has a local max of (-1,1).

a) Write the domain and range in interval notation.
\( P: (-\infty, \infty) \quad R: (-\infty, \infty) \)

b) Find the x-intervals when the function is increasing, decreasing, and constant.
Inc: \( (-\infty, -1) \cup (1, \infty) \)
Dec: \( (-1, 1) \)
Constant:

c) Find the average rate of change from \( x = -1 \) to \( x = 1 \).
\[ m = \frac{0 - (-1)}{1 - (-1)} = \frac{-1}{2} = \frac{-1}{2} \]
6. Given the graph of a function:

   a) Write the domain and range in interval notation.
   \[ D: (-\infty, 9) \quad E: \mathbb{R} \cup [-1, \infty) \]

   b) Find the x-intervals when the function is increasing, decreasing, and constant.
   \[ \text{Inc: } (-5, -2) \text{ Dec: } (-\infty, -6) \cup (-2, 3) \cup (6, 9) \]

   c) Find \( f(0), f(2), f(-2) \).
   \[ f(0) = 5 \quad f(2) = -2 \quad f(-2) = 6 \]

   d) Find the average rate of change from \( x = -1 \) to \( x = 1 \).
   \[ m = \frac{4.5 - 5.5}{1 - (-1)} = \frac{-1}{2} \]

7. Explain how the graph of a polynomial function can help you factor the polynomial.
   The zeroes you see on a graph are the factors (with opposite signs).

8. A student explains that 1, 2, 3, and 4 are the zeros of a cubic polynomial function. Explain why the student is mistaken. There can't be 4 zeroes of a cubic (3rd degree) function.

9. Write an equation of a polynomial function that has three turning points and end behavior that is up and up.
   \[ f(x) = (x + 6)(x + 3)(x - 1)(x - y) \]

Solve by completing the square:

10. \( x^2 + 10x - 3 = 0 \)
    \[ x^2 + 10x + 25 = 3 + 25 \]
    \[ (x + 5)^2 = 28 \]
    \[ x + 5 = \pm 2\sqrt{7} \]
    \[ x = -5 \pm 2\sqrt{7} \]

12. \( x^2 + 20x + 104 = 0 \)
    \[ x^2 + 10x + 100 = -104 + 100 \]
    \[ (x + 5)^2 = -25 \]
    \[ x + 5 = \pm 5i \]
    \[ x = -5 \pm 5i \]

Write the following equations in vertex form (no calculators):

13. \( y = x^2 - 6x + 11 \)
    \[ y = (x - 3)^2 - 2 \]

14. \( y = x^2 - 2x - 9 \)
    \[ y = (x - 1)^2 - 10 \]

15. \( y = x^2 + 16x + 14 \)
    \[ y = (x + 8)^2 - 50 \]
Arithmetic Sequences

Recursive Formula:

\[
\begin{align*}
a_1 &= \text{start} \\
a_n &= a_{n-1} + d 
\end{align*}
\]

\[d = \text{the common difference}\]

\[n = \text{the term number}\]

\[a_n = \text{the } n\text{th term in the sequence}\]

\[a_1 = \text{the 1st term in the sequence}\]

16. Decide whether the following sequences are arithmetic. Explain why or why not.

a. \[7, 12, 17, 22, 27, \ldots\]  
   \[\text{yes! common difference}\]

b. \[3, 6, 12, 24, 48, \ldots\]

17. Write an explicit and recursive rule for the \(n\)th term of the arithmetic sequence. Then find \(a_{25}\).

a. \[1, 5, 9, 13, 17, \ldots\]  
   \[d = 4\]  
   \[\varepsilon: \ 1 + 4(n-1)\]  
   \[a_{25} = 97\]  
   \[\varepsilon: \ 5a_1 = a_{n-1} + 4\]  
   \[\sum a_n = a_{n-1} + 4\]

b. \[3, 7, 11, 15, 19, \ldots\]  
   \[d = 4\]  
   \[\varepsilon: \ 3 + 4(n-1)\]  
   \[a_{25} = 99\]  
   \[\varepsilon: \ 5a_1 = a_{n-1} + 4\]  
   \[\sum a_n = a_{n-1} + 4\]

c. \[-1, 4, -9, 14, -19, \ldots\]
   \[d = -5\]
   \[\varepsilon: \ 1 - 5(n-1)\]
   \[a_{25} = -119\]
   \[\varepsilon: \ 5a_1 = a_{n-1} - 5\]
   \[\sum a_n = a_{n-1} - 5\]

18. Write the first 5 terms of the arithmetic sequences.

a. \[a_n = -2 + 8n\]
   \[6, 14, 22, 30, 38\]

b. \[a_n = 7 - 3n\]
   \[4, 1, -2, -5, -8\]

c. \[a_n = -10 + 4n\]
   \[-6, -2, 2, 6, 10\]

d. \[\begin{align*}
a_1 &= 2 \\
a_n &= a_{n-1} + 4 
\end{align*}\]
   \[2, 6, 10, 14, 18\]

e. \[\begin{align*}
a_1 &= -3 \\
a_n &= a_{n-1} + 2 
\end{align*}\]
   \[-3, -1, 1, 3, 5\]

f. \[\begin{align*}
a_1 &= 5 \\
a_n &= a_{n-1} - 3 
\end{align*}\]
   \[5, 2, -1, -4, -7\]