Factoring $ax^2+bx+c$

Back in the day, we had to use trial and error

Trial + Error

1.) $3x^2+5x+2$
   \[
   (3x + 1)(x + 2) \quad (3x + 2)(x + 1)
   \]
   \[
   3x^2 + 7x + 2 \quad 3x^2 + 5x + 2 \checkmark
   \]

2.) $2x^2 + 7x + 3$
   \[
   (2x + 1)(x + 3)
   \]
   \[
   2x^2 + 7x + 3
   \]
Trial + Error

3.) \(8x^2 - 14x - 15\)

It could be:

\[(8x + 1)(x - 15)\] \[(8x + 3)(x - 5)\]
\[(8x - 1)(x + 15)\] \[(8x - 3)(x + 5)\]
\[(8x + 15)(x - 1)\] \[(8x + 5)(x - 3)\]
\[(8x - 15)(x + 1)\] \[(8x - 5)(x + 3)\]
\[(4x - 1)(2x + 15)\] \[(4x + 5)(2x - 3)\]
\[(4x + 1)(2x - 15)\] \[(4x - 5)(2x + 3)\]
\[(4x + 15)(2x - 1)\] \[(4x - 3)(2x + 5)\]
\[(4x - 15)(2x + 1)\] \[(4x + 3)(2x - 5)\]
Factor By Grouping

• group pairs of terms that have a common factor
• factor out the common factor from each pair
• the resulting parenthesis should be the same
• write the answer as two parenthesis - the common parenthesis times sum or difference of the common factors
Examples

A. Factor \( x^3 - 2x^2 - 7x + 14 \)

\[
\frac{x^2(x-2)}{(x-2)} - \frac{7(x-2)}{(x-2)}
\]

\[
= \frac{x^2-7}{x-2}
\]

B. Factor \( x^3 + 3x^2 + 10x + 30 \)

\[
\frac{x^2(x+3)}{(x+3)} + \frac{10(x+3)}{(x+3)}
\]

\[
= \frac{x^2+10}{x+3}
\]
To use Factor By Grouping for Trinomials where $a \neq 1$

- Multiply $a$ times $c$
- Find the factors of $ac$ that add up to $b$
- Rewrite the expression by replacing the middle term with the sum of the two factors times $x$
- Factor By Grouping
- FOIL to check

NOTE: Some quadratic equations can't be factored

Examples

C.) $3x^2 - 17x + 10 - 15x - 2$  
$\begin{array}{c}
\frac{30}{17} \\
3x(x-5) - 2(x-5) \\
(x-5)(3x-2) \\
3x^2 - 2x - 15x + 10 - 17x
\end{array}$

D.) $4x^2 - 4x - 3$  
$\begin{array}{c}
-6 \\
2x(2x-3) + 1(2x-3) \\
(2x-3)(2x+1) \\
4x^2 + 2x - 6x - 3
\end{array}$
California Method used for factoring $ax^2 + bx + c$

1. Multiply a times c
2. Find factors of ac that add up to b
3. Set up 2 fractions with $ax$ in the numerator and each factor of ac in the denominator
4. Reduce the fractions
5. Set up parenthesis (num+denom)(num+denom)
6. FOIL to check.

Note: Some quadratic equations can’t be factored.

E.) $3x^2 - 17x + 10$
\[
\frac{30}{15} \times \frac{-2}{-17}
\]
\[
\frac{3x}{-15} \quad \frac{3x}{-2} \quad \frac{3x}{-5} \quad \frac{3x}{-2}
\]

$(x-5)(3x-2)$

F.) $4x^2 - 4x - 3$
\[
\frac{12}{-6} \times \frac{-4}{-4}
\]
\[
\frac{4x}{-6} \quad \frac{4x}{2} \quad \frac{2x}{-3} \quad \frac{2x}{1}
\]

$(2x-3)(2x+1)$